Journal of Engineering Physics and Thermophysics, Vol. 75, No. 2, 2002

CHAOTIC HOMOGENEOUS POROUS MEDIA. 2. THEORY OF DISPERSION TURBULENCE: BASIC PROPOSITIONS

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UDC 536.24:544.77

The basic propositions of the theory of dispersion turbulence — the principles of quasi-one-dimensionality, independence, and isotropy, random velocity fields, and local laws and coefficients of transfer — are presented.

Macrodispersion processes in inhomogeneous porous media govern the steady physical state of the system solid body–liquid (the liquid can be replaced by a gas).

In the past fifty years, a vast number of publications devoted to the dispersion and inhomogeneity of porous structures have appeared, but only some of them have been distinguished by new ideas and approaches. The origin of dispersion theory is associated with the name of Taylor and with the investigations of the Cambridge school (G. I. Taylor, R. Aris, and P. G. Saffman) [1–5]. The diffusion coefficient for Poiseuille flow in a capillary tube was calculated and the first model of dispersion in a porous medium based on the scheme of independent random walks of liquid moles, i.e., the analog of Brownian motion, was constructed. A. E. Scheidegger presented in [6] not only a review of the most important works on the physics of filtration over a hundred years, which makes the book comparable to the fundamental work [7] in this respect, but also put forward the idea that hydrodynamics and Darcy's law are largely determined by dispersion effects. V. N. Nikolaevskii, extending the theory, proposed the general form of the dispersion tensor [8, 9] which is considered to be conventional at present: the dispersion coefficients are in proportion to the first power of the flow velocity. By the 1990s, dispersion theory had gradually acquired two pronounced trends, which do not always complement each other. One trend corresponds to consideration of dispersion at the microlevel: the statistics of different configurations of pores and of their size and orientation in space is considered. From the prescribed probability characteristics of the geometric models of a porous medium one determines the coefficients of transfer and dispersion. Examples of this trend in investigations are [10–12].

Another extension of the theory is associated with consideration of a porous medium at the macrolevel, where the set of values of physical parameters is presented as one realization of a random process. This trend has occurred owing to the classical results of the works of A. N. Kolmogorov, the outstanding mathematician of the XXth century, and his disciples on the theory of random processes and turbulence [13, 14]. The theory of steady-state random processes was used by M. I. Shvidler [15–17] and Yu. A. Buevich [18–21] as applied to the filtration problems in porous media. By introducing the random fields of permeability, pressure, velocity, and temperature and postulating that Darcy's law and the laws of heat exchange and mass exchange hold true for such fields, one can obtain a system of stochastic equations which is easy to solve. But, first, Darcy's law and others have been established experimentally for the average values. Second, the stochastic models used are described on the basis of the available empirical data only approximately; for example, one has to accept the hypotheses of the correlation scale, the form of the correlation function, and the character of the functional dependence of the permeability on the porosity. Thus, consideration at the macrolevel requires no less experimental information for its practical implementation than the first trend. Despite this fact, an analysis of filtration envisaged as a random process is distinguished by mathematical elegance [17]; some indisputable results of this model will be used in the theory of dispersion turbulence.

Two more problems associated with the description of transfer processes, which none of the thorough reviews [16, 22, 23] eliminates, exist. These are the problems of averaging and computation of the effective coefficients of conductivity of porous systems.

No general theory of averaging of embedded continua with solution of ergodic problems has been created, but a number of interesting results which predict its thorough development in the future have been obtained [24–31]. In

"Algoritm-M" Joint-Stock Association, Moscow, Russia; email: mog@mail.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 2, pp. 85–95, March–April, 2002. Original article submitted May 10, 2001.

the theory of dispersion turbulence, we will use the most simple and clear methods of averaging, the main criterion of correctness of which is the agreement between the results of solution of the averaged equations and the experimental data. Such a method has successfully been employed in [32].

The theory of determination of the effective coefficient of conductivity of heterogeneous media has a century-old history of development and is essentially an independent scientific trend. In what follows we will assume that the coefficient of transfer of a stationary heat-transfer agent and of a moving one in a porous media are independent and additive; the stationary component is computed from the ready formulas obtained in the theory of effective conductivity with allowance for specific features of the problem. In this connection, we briefly touch upon the development of the methods to determine the effective conductivity of porous media, singling out just certain fundamental works. The problem of computation of the effective parameters of inhomogeneous systems was posed for the first time in the works of Poisson, who studied the magnetic properties of inhomogeneous media with inclusions. In more recent times, Mossotti and then Clasius used the Poisson method to study inhomogeneous dielectrics. Solution of problems of such type in optics is associated with the names of Lorenz and Lorentz, who studied the refractive indices of media as functions of the polarizability and concentration of particles. In the works of Maxwell and Rayleigh, consideration has been given to the problem of conductivity of matrix-type systems with inclusions of another conductivity which are arranged regularly or chaotically, and formulas correct in the approximation of low concentration of the inclusions have been obtained for the effective conductivity of such systems. The approach developed by Maxwell and Rayleigh gave rise to numerous publications in which different partial problems have been considered and asymptotic formulas for the effective conductivity have been obtained. One can study some of them in the fundamental handbook of A. V. Luikov [33].

One method of calculation of the effective characteristics of inhomogeneous media is based on the theory of self-consistent fields initially developed in quantum mechanics and related to the analysis of multifrequency interactions. The principle of self-consistency implies that in calculating the field inside the inclusion it is considered to be surrounded by an "effective" medium, i.e., a medium whose conductivity is identical to the sought effective conductivity. By averaging the field calculated in such a manner over all the inclusions and setting it equal to the prescribed macroscopic field, we obtain the equation for finding the effective conductivity. It is probable that the first self-consistent parameters were computed by D. A. G. Bruggeman [34]. Consideration of the problem on finding the self-consistent field in a complete formulation is associated with the use of the perturbation theory, i.e., the basic tool of modern theoretical physics. Upon selection of the corresponding "unperturbed" problem, the solution is written in the form of a series in a certain parameter, i.e., a perturbation interpreted rather broadly at present. The main difficulty involves summation of the series obtained, which is impossible to carry out completely as a rule. Therefore, one has to confine oneself to one approximation or another; this gives rise to numerous formulas to calculate the effective parameters. Attempts at summing up the entire series of the perturbation theory or at least accelerating its convergence are associated with the renormalization technique. Analogous results can be obtained without writing the series of perturbations if the singular and formal derivatives of the Green function in the basic functional equation are separated. These methods are similar to the methods of approximate summation of a perturbation series written for the Fourier amplitudes of fluctuating fields and of summation using the diagram technique of Feynman. One can study the self-consistent method in [17, 35, 36] in sufficient detail.

The development of the methods of percolation theory made it possible to solve numerically a number of problems of determination of the effective conductivity of inhomogeneous plane and spatial lattice structures by the Monte Carlo method. The results are given in [17, 37, 38] and are in good agreement with self-consistent solutions.

The specific interests of percolation theory are associated with the behavior of inhomogeneous systems in the vicinity of the point of semiconductor–insulator transition and with study of the topology of conducting and nonconducting regions i.e., infinite clusters. In particular, the behavior of generalized conductivity (dielectric permittivity and magnetic permeability, viscosity, electrical conductivity, thermal conductivity, coefficients of diffusion and filtration, elastic moduli) near the threshold in such systems are described by scaling-type power laws [39–43].

The problem of computation of the effective characteristics of inhomogeneous media also allows a variational formulation which enables one to pose problem on limits within which the effective characteristics of a certain class of systems are confined, in other words, to construct the range for the exact value of the effective characteristic. It is clear that the range will be the narrower the more information on the system in question, more precisely, on the class of systems to which it belongs. Thus, if the class of the media in question is not limited, i.e., no additional information is used, the variational limits yield a universal range: the effective conductivity of any medium is confined between the harmonic mean and arithmetic mean conductivities. It is probable that this range was established for the first time in [44], for the moduli of elasticity and compliance in [45, 46], and for conductivity in [47] by A. M. Dykhne. Of great interest is the Haschin–Strikman range [48] for the effective conductivity of macro- and microscopically isotropic multiphase systems. This range is much narrower than the universal one. Furthermore, according to [49], the Haschin–Strikman range is appropriate for two-dimensional systems and plane two-phase isotropic systems exist whose conductivity coincides with the limits of the range. Having the ranges at one's disposal, one can construct approximate solutions.

In this article, it is impossible to comment in detail on the remaining classes of works on determination of the effective generalized conductivity of heterogeneous systems. Therefore, we confine ourselves to just make reference to them. First of all, these are the detailed works of G. N. Dul'nev and co-workers on structures with interpenetrating components and on percolation theory [50–59], the classical work of V. I. Odelevskii on the conductivity of matrix systems [60], and the mathematical investigations of effective conductivity of A. M. Dykhne [61], Yu. A. Buevich [62–64], and M. I. Shvidler [65, 66].

In engineering thermal physics, the entire class is occupied by the methods of structural modeling, the beginning and development of which is associated with the monograph of A. F. Chudnovskii [67]. The actual porous structure is modeled by the most appropriate ordered structure in which one singles out either a unit cell or a period for which the thermal conductivity is calculated [28, 51, 52, 67, 68].

Information on semiempirical and empirical (most reliable) methods of determination of the effective conductivity can be obtained from the proceedings of the All-Union seminar [69].

Thus, a wide choice of calculation-theoretical and experimental dependences to determine the coefficients of generalized conductivity of inhomogeneous porous media exists.

A few words about fractal theory. This mathematical theory has successfully been developed in recent times, mainly in physical applications [70–83]. As has been indicated above, the distinctive features of the behavior of the processes occurring in porous media are attributed to percolation effects but, as is well known (see, for example, [41]), a percolation cluster has fractal properties. There can be the cases where the fractal is the pore space, the solid skeleton, or the interior surface of a porous system. In this connection, a number of interesting works have appeared where fractal theory is used to describe the structure of porous media [84–86], heat conduction [85, 88], filtration [86, 87, 89], and heat exchange [66, 87]. But we have another important aspect which possibly relates fractal theory to the presented *theory of dispersion turbulence*. As will be shown, the theory of dispersion turbulence is a *closed statistical-phenomenological theory of transport in inhomogeneous porous media* which requires no heuristic considerations associated with energy transitions of different scales for its construction (this property is a distinguishing feature of the very interesting empirical model of V. V. Kharitonov which relates the hydraulic resistance and the coefficient of internal heat transfer of a porous medium [90–93]). In the theory of dispersion turbulence, the large-scale steady-state fluctuations of the temperature and velocity fields which are attributed to the inhomogeneity of a porous structure are of prime importance in heat exchange and hydrodynamics. This raises the question: "Is there a transfer of energy from the small-scale pulsations in the pores to the large-scale pulsations of dispersion turbulence?" This transfer of energy in ordinary two-dimensional turbulence is well known; it is called the inverse cascade, whose very existence is attributed to the fractal character of turbulence [94–100]. It is obvious that the existence of an inverse cascade in inhomogeneous porous media is no more than a hypothesis necessitating a thorough analysis and investigation in the regimes of developed turbulent flow.

In this work, we do not comment on the results of experimental investigations of heat and mass exchange in porous media: this necessitates separate consideration. Here we only mention the monograph [101], the experimental data of which provide a basis for many theoretical works.

The basic regularities of the theory of dispersion turbulence were established by the author nearly a decade ago with the example of solution of the heat-exchange problem of porous cooling. The procedure was developed [10], a setup was created [105], and the results of experimental investigation of the internal heat exchange and hydrodynamics in substantially inhomogeneous powder sintered porous specimens of stainless steel [102] were obtained earlier and independently at the Moscow Aviation Institute. The comparison of the experimental data to the conclusions of a theoretical analysis confirmed the correctness of the solution [111–115]. But the theory was presented in distorted form, since the quantities used in processing the experimental data, i.e., $\bar{d}_{\text{pore}}$, average diameter of the pores, $\bar{d}_{\text{part}}$, average diameter of the particles, and the inhomogeneity number $\bar{\sigma}$ [106, 108–110], are conventional and nonrigorously defined. They are not used in the theory of dispersion turbulence. We were forced to change the theory and to make it cumbersome [113] in order to pass from these quantities to a dispersion diameter d_D, i.e., the linear dimension which is simultaneously the internal scale and the parameter characterizing the inhomogeneity of the medium [116, 117]. This parameter is determined and preserves its significance for systems with interpenetrating components, infinite clusters, and fractal formations for which $\bar{d}_{\text{pore}}$, $\bar{d}_{\text{part}}$, and $\bar{\sigma}$ lose their meaning. The dispersion diameter is easily changed. The porous specimens were transferred to the I. P. Bardin Central Scientific-Research Institute of Ferrous Metallurgy and were structurally tested on a Kvantimet-720 TV microscope. The hypothesis of normal distribution (basic structural theorem [116, 117]) was confirmed from the histogram of porosity distribution at a significance level of 0.05 according to the Pearson criterion. The average porosity $\overline{\Pi}$, the dispersion $D[\Pi]$, and the dispersion diameter d_D were determined for each specimen. These quantities are simpler in determination than $\bar{d}_{\text{pore}}$, $\bar{d}_{\text{part}}$, and $\bar{\sigma}$. The correct structural analysis, the extension of the theory to hydrodynamics, and the high accuracy of the confirming experiments (the error was 15% for the internal heat exchange and 5% for the hydraulic resistance [108]) make the theory of dispersion turbulence sufficiently reliable. Moreover, in other well-known works on investigation of heat and mass exchange, there are experimental data which also confirm the theory.

The theory will be presented without excessive mathematical formalism but correctly and in a form convenient for thermophysical engineering experimenters. The results of the theory of dispersion turbulence can successfully be used to solve practical problems, for example, of thermal protection [118, 119], and as the additional model of internal heat exchange for the classical theory of S. S. Kutateladze and A. I. Leontiev [120–125]. The theory of dispersion turbulence can also be applied to calculations of reactors with chaotically arranged fuel elements [126–129] and chemical reactors with a granular bed [32, 101].

Basic Propositions of the Theory of Dispersion Turbulence. *Dispersion* means the scattering and transfer of any substance (mass, electric charges, field strength, energy, momentum, etc.) in a porous medium. This is a general definition. In [10, 33], only the transfer of mass is meant by dispersion. The extension of the concept is associated with the fact that the effective conductivity of heterogeneous structures, the effective thermal conductivity of the heat-transfer agent in them, etc. are determined by scattering. Nine mechanisms of dispersion mass transfer, i.e., molecular diffusion, migration of turbulent vortices, crookedness, self-correlation of the trajectories of liquid moles, recirculation, effects of dead-end pores, adsorption, hydrodynamic mass transfer in a pore, and macrodispersion, are presented in [10]. A. V. Luikov added here diffusion of slip in a pore [33]. This list can be supplemented with even more subtle but insignificant effects. Dispersion mechanisms associated with the overall action of a large number of pores, in particular, macrodispersion, is the subject of investigation of the theory of dispersion turbulence. The nonrigorous definition of this phenomenon in [10] ("...macrodispersion is attributed to different deviations from an ideal porous medium which cause the distortion of general streamlines") is unsatisfactory. We introduce the following definition in advance.

Dispersion turbulence means the scattering of a moving liquid on porosity inhomogeneities. The concept of inhomogeneity in chaotic porous structures is determined by the *principal theorem* in [117].

T h e o r e m. *In a chaotic inhomogeneous, homogeneous, and isotropic porous medium, the distribution density of the porosity for an arbitrary area S of any cross section obeys the normal law*

$$W(\Pi) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left[-\frac{\Pi - \overline{\Pi}}{2\sigma^2}\right], \quad \sigma^2 = D[\Pi] = \overline{\Pi}\,(1 - \overline{\Pi})\,\frac{S_D}{S}\,, \tag{1}$$

where $S_D = d_D^2$, d_D is the *dispersion diameter*, the basic linear characteristic of the inhomogeneity of a homogeneous porous structure (here it is more convenient to use this definition of a dispersion diameter whose value is $\sqrt{4/\pi} \approx 1.1$ times lower than in [116, 117]) for the square S with side d_S in the distribution (1) $\sigma^2 = D[\Pi] = \overline{\Pi}(1 - \overline{\Pi})/d_S^2$. The analogous normal law holds true for the porosity distribution over the volume.

It is probable that the normal law of distribution of the porosity over the volume was mentioned for the first time in the work of Collins [130]. The mathematical constructions given in the monograph are not a proof since they are incorrect. But the final result is correct. Moreover, Collins gave experimental data which indicate the strong dependence of the permeability of sandstone on the structure parameter determining the dispersion of the porosity distribution, i.e., "the characteristic volume ε_0." It is clear that this volume is proportional to the third power of the dispersion parameter d_D, i.e., the dependence of the permeability on the linear characteristic of the inhomogeneity is even more pronounced.

Dispersion turbulence is a steady-state phenomenon or a quasi-steady-state one if the dependence of the thermophysical parameters of the liquid and the skeleton on the temperatures in unsteady regimes of heating or cooling is taken into account. The term "dispersion turbulence" has been introduced by the author following Taylor, who noted that "... the motion of a liquid in a porous medium is frozen turbulence" [3]. Apart from this name we will use the equivalent one (the same as in [10]): *macrodispersion*. For a correct mathematical determination of dispersion turbulence (macrodispersion) and its properties, it is proposed that the following *three basic principles* be observed.

1. Principle of quasi-one-dimensionality. For the average rate of filtration v along the Z axis, the velocity of the liquid for the area $S = \Delta x \Delta y$ (Fig. 1) is the random quantity $v_z(S) = v \dfrac{\Pi(S)}{\overline{\Pi}}$ with the normal law of distribution

$$W(v_z) = \frac{1}{\sqrt{2\pi}\,\sigma_z} \exp\left[-\frac{(v_z - v)^2}{2\sigma_z^2}\right], \tag{2}$$

where $\sigma_z^2 = D[v_z(S)] = \dfrac{1-\overline{\Pi}}{\overline{\Pi}}\left(\dfrac{v d_D}{\Delta L}\right)^2$, $\Delta L = \Delta x = \Delta y = \Delta z$.

Having introduced the spatial pulsations $v_z(S) = \overline{v}_z + v_z' = v + v_z'$, $v_x(S) = v_x'$, $v_y(S) = v_y'$, and $\overline{v}_x = \overline{v}_y = 0$, we obtain the normal law of distribution for v_z':

$$W(v_z') = \frac{1}{\sqrt{2\pi}\,v_\parallel} \exp\left[-\frac{1}{2}\left(\frac{v_z'}{v_\parallel}\right)^2\right]. \tag{3}$$

For the component of the root-mean-square velocity of dispersion turbulence which is in parallel to the main flow, we have

$$v_\parallel = \sqrt{\frac{1-\overline{\Pi}}{\overline{\Pi}}}\,\frac{v d_D}{\Delta L}. \tag{3a}$$

The principle of quasi-one-dimensionality has a clear meaning. First, the well-known hypothesis of Dupuit and Forchheimer is used for the liquid velocity in a pore space; second, the porosity distribution over the cross section is determined by the normal law according to the principal structural theorem. Therefore, by virtue of the ergodicity of the average values of the porosity, the average values of the velocities over the ensemble and the area will also be equal.

2. Independence principle. Apart from the average rate of filtration v, use is often made of the average specific mass flow rate $\dot{m} = \rho v$ and the average flow rate over the area $\dot{M} = \dot{m}S = \rho v S$. By analogy with these constant parameters we consider the random quantities $\dot{m}_\alpha(S) = \rho v_\alpha(S)$ and $\dot{M}_\alpha = \dot{m}_\alpha S = \rho v_\alpha S$, where α determines the directions of the components along the axes $\alpha = \{x, y, z\}$. Let there be the *independence of the flow rates for parallel cross sections in any direction*, i.e., cov $\{v_\alpha(S), v_{\alpha+\Delta L}(S)\}$ = cov $\{\dot{m}_\alpha(S), \dot{m}_{\alpha+\Delta L}(S)\}$ = cov $\{\dot{M}_\alpha(S), \dot{M}_{\alpha+\Delta L}(S)\} = 0$ *for all* $\Delta L \geq R$. This principle has a simple explanation. Dispersion turbulence (macrodispersion) manifests itself in volumes with linear dimensions exceeding the correlation radius. For example, for chaotic spherical packings the maximum correlation radius is only 4 to 5 diameters of the particles [131]. Furthermore, a scale at least one order of magnitude larger than the size of the particles or the pores is required for correct averaging [33]. On the one hand, the independence principle simplifies an analysis of dispersion turbulence (all two-point, second-rank correlation tensors are identically equal to zero), and on the other, complicates it because of the impossibility of the limiting transition for $\Delta L \to 0$.

3. Principle of isotropy. If in a chaotic, homogeneous, and isotropic porous medium there is a uniform flow of a liquid (direction of the flow is constant), the random velocity fields of dispersion turbulence will be invariant relative to the rotations about the direction of the average velocity and mirror reflections relative to the planes which are perpendicular to the velocity vector. Therefore, all single-point tensors linearly related to the velocity field and reduced to the principal axes have a diagonal form (of diffusion, thermal conductivity, etc.). The principle of isotropy for porous structures was introduced by V. N. Nikolaevskii in 1959 [8].

To define the distribution functions $v_x'(S)$ and $v_y'(S)$ we use the continuity equation for the cube $\Delta x = \Delta y = \Delta z = \Delta L$ in a porous medium (Fig. 1). We introduce the notation $\Delta\dot{M}_z = \dot{M}_{z+\Delta l} - \dot{M}_z$, $\Delta\dot{M}_x = \dot{M}_{x+\Delta L} - \dot{M}_x = \Delta\dot{M}_x'$ ($\dot{M}_x = \dot{M}_x'$, $\dot{M}_{x+\Delta L} = \dot{M}'_{x+\Delta L}$), $\Delta\dot{M}_y = \dot{M}_{y+\Delta L} - \dot{M}_y = \Delta\dot{M}_y'$ ($\dot{M}_y = \dot{M}_y'$, $\dot{M}_{y+\Delta L} = \dot{M}'_{y+\Delta L}$).

Then the continuity equation will be obtained in the form

$$\Delta\dot{M}_z + \Delta\dot{M}_x' + \Delta\dot{M}_y' = 0\ .$$

Using the principle of isotropy $\Delta\dot{M}_x' = \Delta\dot{M}_y'$ we find

$$\Delta\dot{M}_z = -\,2\Delta\dot{M}_x' = -\,2\Delta\dot{M}_y'\ .$$

The random quantities $\Delta\dot{M}_x'$, $\Delta\dot{M}_y'$, $\dot{M}_x'(S)$, $\dot{M}_y'(S)$, $v_x'(S)$, and $v_y'(S)$ have the normal laws of distribution, since $v_z(S)$, $\dot{M}_z(S)$, and $\Delta\dot{M}_z$ have such a distribution according to the principle of quasi-one-dimensionality. Having taken the square of the last equalities and averaging with the use of the independence principle, we obtain

$$D\,[\dot{M}_z\,(S)] = 4D\,[\dot{M}_x'\,(S)] = 4D\,[\dot{M}_y'\,(S)]$$

or for the velocities

$$D\,[v_z\,(S)] = D\,[v_z'\,(S)] = 4D\,[v_x'\,(S)] = 4D\,[v_y'\,(S)]\ .$$

Then for the component $v_x'(S)$ with account for (3a) the distribution

$$W\,(v_x') = \frac{1}{\sqrt{2\pi}\,v_\perp}\exp\left[-\frac{1}{2}\left(\frac{v_x'}{v_\perp}\right)^2\right],\quad v_\perp = \frac{1}{2}\sqrt{\frac{1-\overline{\Pi}}{\overline{\Pi}}\,\frac{vd_D}{\Delta L}}\ . \tag{4}$$

holds true. We have the analogous distribution for the component $v_y'(S)$:

$$W\,(v_y') = \frac{1}{\sqrt{2\pi}\,v_\perp}\exp\left[-\frac{1}{2}\left(\frac{v_y'}{v_\perp}\right)^2\right],\quad v_\perp = \frac{1}{2}\sqrt{\frac{1-\overline{\Pi}}{\overline{\Pi}}\,\frac{vd_D}{\Delta L}}\ . \tag{5}$$

The ratio of the longitudinal component of the root-mean-square velocity of dispersion turbulence to the transverse component is $v_\parallel/v_\perp = 2$, which corresponds to the estimates obtained by M. A. Gol'dshtik from other considerations [32].

Distributions (3)–(5) determine the *random field of velocities of dispersion turbulence*. Now we can give a more rigorous definition of dispersion turbulence. *Dispersion turbulence (macrodispersion)* means the steady-state chaotic motion of a liquid in a porous medium attributed to the scattering of the main flow on porosity inhomogeneities with the random Gaussian velocity field (3)–(5).

We calculate the Reynolds numbers of dispersion turbulence for regions in the porous medium with an arbitrary characteristic linear dimension d_S (circle $S = \pi d_S^2$, square $S = d_S^2$); by using (3a) and (4), for $\Delta L = d_S$ we obtain

$$\mathrm{Re}_\parallel = \frac{v_\parallel d_S}{\nu} = \sqrt{\frac{1-\overline{\Pi}}{\overline{\Pi}}\,\frac{vd_D}{\nu}}\,,\quad \mathrm{Re}_\perp = \frac{v_\perp d_S}{\nu} = \frac{1}{2}\sqrt{\frac{1-\overline{\Pi}}{\overline{\Pi}}\,\frac{vd_D}{\nu}}\ .$$

Thus, the Reynolds number does not depend on the linear dimension of the region; d_D is the characteristic scale of dispersion turbulence. Moreover, for example, for spherical packings $\sqrt{(1-\overline{\Pi}/\overline{\Pi}} \approx 1.2$ and $d_D \approx d_{\text{eq}}$ [116, 117]; therefore, the Reynolds number usually used to describe hydrodynamics and heat exchange is $\text{Re}_{\text{eq}} = vd_{\text{eq}}/\nu \approx \text{Re}_{\text{II}}$. Needless to say, this fact is not a proof of the dispersion turbulence determining the processes of heat and mass exchange in porous media, but it is remarkable.

Local Laws of Transfer. Dispersion turbulence is the chaotic motion of flows of a liquid. Consequently, laws analogous to the laws of Fourier, Newton, and Fick with the corresponding coefficients of thermal conductivity, viscosity, and diffusion must hold true for dispersion turbulence [33, 132, 133]. These coefficients are easy to determine. As previously, the geometric object of investigation is the cube $\Delta x = \Delta y = \Delta z = \Delta L$ which is arbitrary singled out in a porous medium (Fig. 1); the Z axis is in parallel to v.

1. Macrodispersion diffusion (self-diffusion). Let us assume initially that in the lower half of the cube the concentration of a substance introduced into the main flow is c_x, while in the upper half the concentration is $c_{x+\Delta L}$. For definiteness we take $c_x > c_{x+\Delta L}$. Then the transfer of the substance from the bottom upward through the area $S = (\Delta L)^2$ of the central cross section of the cube which is perpendicular to the X axis is determined by the average dispersion flow $\dot{M}_{x,c} = c_x v_\perp (\Delta L)^2 \overline{\Pi}$ and the transfer from the top downward is determined by the flow $\dot{M}_{x+\Delta L,c} = c_{x+\Delta L} v_\perp (\Delta L)^2 \overline{\Pi}$. The resultant diffusion flow is equal to

$$\Delta\dot{M}_{x,c} = \dot{M}_{x,c} - \dot{M}_{x+\Delta L,c} = -\frac{1}{2}\sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; vd_D \frac{\Delta c_{\Delta L}}{\Delta L} (\Delta L)^2 , \tag{6}$$

where $\Delta c_{\Delta L} = c_{x+\Delta L} - c_x$.

The concentration cannot change abruptly, but the initial assumption can easily be abandoned if we note that Eq. (6) preserves its form for all the cubes $\Delta L \geq \Delta L_1 \geq \Delta L_2 \geq \ldots \geq 0$ embedded into one another in the case of preservation of the ratios

$$\frac{\Delta c_{\Delta L}}{\Delta L} = \frac{\Delta c_{\Delta L_1}}{\Delta L_1} = \frac{\Delta c_{\Delta L_2}}{\Delta L_2} = \ldots = \frac{dc_x}{dx} . \tag{7}$$

Formally, equalities (7) are extended to the case $\Delta L \to 0$ for the most natural and simple introduction of a derivative. But the dispersion turbulence is determined for volumes with linear dimensions $\Delta L \geq R$. This difficulty is surmountable: the derivative dc_x/dx can be determined on the basis of the principle of compressive stresses without a limiting transition for $\Delta L \to 0$ [134]. Therefore, from (6) and (7) we obtain

$$\Delta\dot{M}_{x,c} = -\frac{1}{2}\sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; vd_D \frac{dc_x}{dx} S . \tag{8}$$

Equation (8) is the first Fick law with the diffusion coefficient $D_{M,x}$:

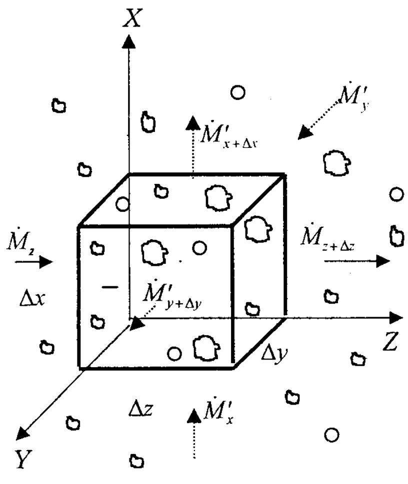
$$D_{M,x} = \frac{1}{2}\sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; vd_D . \tag{8a}$$

The Fick law is analogously proved for the remaining components of the coefficient of dispersion diffusion:

$$D_{M,y} = D_{M,x} = D_{M,\perp} = \frac{1}{2}\sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; vd_D , \quad D_{M,z} = D_{M,\parallel} = \sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; vd_D . \tag{9}$$

If the X axis is brought into coincidence with the direction of the main flow, the Nikolaevskii tensor [8] for the dispersion diffusion has the form

$$D_M = \sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; vd_D \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} . \tag{10}$$



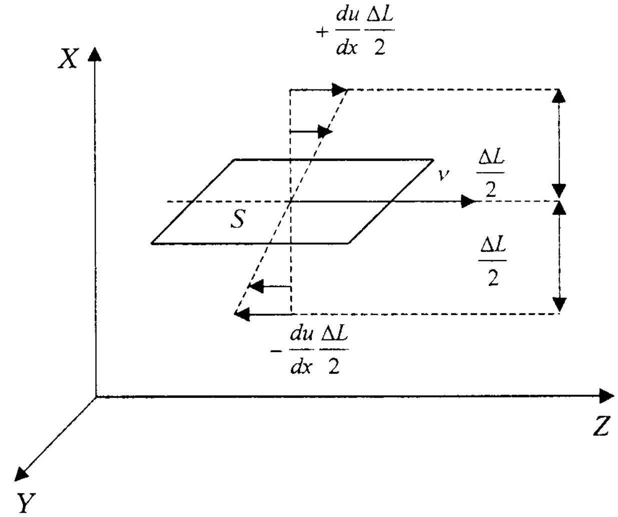


Fig. 1. Determination of the velocity field of dispersion turbulence. $\dot{M}_z$, $\dot{M}_{z+\Delta z}$, $\dot{M}'_x$, $\dot{M}_{x+\Delta x}$, $\dot{M}'_y$, and $\dot{M}'_{y+\Delta y}$, kg/sec; Δx, Δy, Δz, m.

Fig. 2. Computation of the coefficient of dispersion viscosity. ΔL, m; S, m^2; v, m/sec; du/dx, sec^{-1}.

In the absence of filtration in the porous medium we will have molecular diffusion whose coefficient $D_{\text{eff},0}$ can be calculated from the formulas of generalized conductivity, as has been indicated above (for example, from the Maxwell formula $D_{\text{eff},0} = D_m 2\overline{\Pi}/(3 - \overline{\Pi})$ [33]). The total effective coefficients of diffusion are determined by the additivity

$$D_{\parallel} = D_{\text{eff},0} + \sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; v d_D\,, \quad D_{\perp} = D_{\text{eff},0} + \frac{1}{2}\sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; v d_D\,. \tag{11}$$

2. Macrodispersion thermal conductivity. Let the liquid in the lower half of the cube have a higher temperature $T_x > T_{x+\Delta L}$. There is no heat conduction over the solid skeleton. Then the resultant average dispersion heat flux from the bottom upward through the central area $S = (\Delta L)^2$ is equal to

$$Q_x = -\,c_p\,\rho v_{\perp}\,\Delta T_x\,(\Delta L)^2\,\overline{\Pi} = -\frac{1}{2}\sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; c_p\,\rho v d_D\,\frac{\Delta T_x}{\Delta L}\,(\Delta L)^2\,, \tag{12}$$

where $\Delta T_x = T_{x+\Delta L} - T_x$.

Using considerations analogous to the ones above, from (12) we obtain the Fourier law

$$Q_x = -\frac{1}{2}\sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; c_p\,\rho v d_D\,\frac{dT_x}{dx}\,S\,. \tag{12a}$$

The Fourier law will also hold true for the remaining axes with the components of the coefficient of dispersion thermal conductivity

$$\lambda_{M,x} = \lambda_{M,y} = \lambda_{M,\perp} = \frac{1}{2}\sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; c_p\,\rho v d_D\,, \tag{13}$$

$$\lambda_{M,z} = \lambda_{M,\parallel} = \sqrt{\overline{\Pi}\,(1-\overline{\Pi})}\; c_p\,\rho v d_D\,. \tag{14}$$

With allowance for the influence of the molecular thermal conductivity of the liquid, the components will have the form

$$\lambda_{\|} = \lambda_{\text{eff},0} + \sqrt{\overline{\Pi}\,(1 - \overline{\Pi})}\; c_p\,\rho v d_D\,, \quad \lambda_{\perp} = \lambda_{\text{eff},0} + \frac{1}{2}\sqrt{\overline{\Pi}\,(1 - \overline{\Pi})}\; c_p\,\rho v d_D\,. \tag{15}$$

3. Macrodispersion viscosity. We compute the coefficients of apparent viscosity by another method, formally using the Newton law for dispersion flows. Just as for the diffusion and thermal conductivity, we calculate the components of the viscosity coefficient η_x and η_y for the velocity gradients which are perpendicular to the main flow and η_z for the velocity gradients which are parallel to it.

We consider the central cross section of the cube $S = (\Delta L)^2$ and superimpose a laminar field of velocities with a gradient along the X axis on a constant main flow (Fig. 2). According to the Newton law, since v = const the force of internal friction is $F = \eta_{M,x} S \frac{du}{dx}$; on the other hand, the source is equal to the change in the momentum per unit time. Therefore,

$$\eta_{M,x} = \frac{\Delta K}{\Delta t S \dfrac{du}{dx}}\,.$$

It is easy to calculate the total average change in the momentum (Fig. 2):

$$\Delta K = \rho v_{\perp}\, \Delta t S \frac{du}{dx}\, \Delta L\, \overline{\Pi}\,,$$

therefore, $\eta_{M,x} = \rho v_{\perp} \Delta L = \frac{1}{2}\sqrt{\overline{\Pi}(1 - \overline{\Pi})}\,\rho v d_D$. It is clear that

$$\eta_{M,y} = \eta_{M,x} = \eta_{M,\perp} = \frac{1}{2}\sqrt{\overline{\Pi}\,(1 - \overline{\Pi})}\;\rho v d_D\,. \tag{16}$$

Analogously we calculate the coefficient η_z for the longitudinal velocity gradients:

$$\eta_{M,z} = \eta_{M,\|} = \sqrt{\overline{\Pi}\,(1 - \overline{\Pi})}\;\rho v d_D\,. \tag{17}$$

To determine the local coefficients of transfer of macrodispersion we used physically clear methods of analysis of transport phenomena in gases, but formulas (9), (10), (13), (14), (16), and (17) can be derived strictly directly from the random field of velocities (3)–(5). It is more convenient to carry out further refinement and analysis of formulas (11) and (15) in deriving the basic equation of internal heat exchange in inhomogeneous porous media.

The deviation of the local laws of transfer will be completed by equalities which relate the coefficients of transfer of dispersion turbulence:

$$\frac{c_p \eta_{M,\|}}{\lambda_{M,\|}} = \frac{c_p \eta_{M,\perp}}{\lambda_{M,\perp}} = \mathrm{Pr}_M = 1\,, \tag{18}$$

$$\eta_{M,\|} = \rho D_{M,\|}\,, \quad \eta_{M,\perp} = \rho D_{M,\perp}\,. \tag{19}$$

When $\overline{\Pi}$ = 0.5 all the macrodispersion coefficients of transfer for homogeneous media with nonuniform properties $0 \le \Pi < 1$ take on the maximum values. The entropy of the structure for such a porosity also has its extremum [117].

Dispersion turbulence is attributed to the nonuniformity of porosity distribution in space as a result of which the total effective action of microprocesses will be different for individual objects consisting of a great number of pores. These differences lead to the occurrence of a macrodispersion continuum. Dispersion turbulence is a macroscale random process that forms stable and steady-state space structures: the fields of temperatures, velocities, and concentrations [111–115, 127–129].

Macrodispersion effects manifest themselves when the heat- and mass-transfer processes in the system solid skeleton–liquid reach the steady-state regime throughout the space of a porous medium.

The steady-state dispersion turbulence, acting between macrovolumes, exerts a substantial influence on the average parameters of the processes of heat and mass exchange which must be determined with allowance for the regularities presented in this work.

NOTATION

D_m, coefficient of molecular diffusion of the liquid; ρ, density of the liquid; v, coefficient of kinematic viscosity of the liquids; $\lambda_{eff,0}$ and $D_{eff,0}$, effective coefficients of thermal conductivity and diffusion of the stationary liquid in a porous medium; d_{pore} and d_{part}, average diameters of the pores and the particles; $\overline{\sigma}$, inhomogeneity parameter in [110]; d_D and S_D, dispersion diameter and area; S, arbitrary area; σ, standard deviation; ε_0, characteristic volume in [130]; v, velocity of the liquid in a porous medium; Π, porosity; d_S, Δx, Δy, Δz, ΔL, ΔL_1, and ΔL_2, linear dimensions; $\dot{m}$ and $\dot{M}$, specific and total flow rate of the liquid; R, correlation radius of the porous structure; d_{eq}, equivalent diameter; c_x and $c_{x+\Delta L}$, concentrations; $\Delta c_{\Delta L}$, $\Delta c_{\Delta L_1}$, and $\Delta c_{\Delta L_2}$, concentration difference; $\dot{M}_{x,c}$, $\dot{M}_{x+\Delta L,c}$, and $\Delta M_{x,c}$ diffusion flows; D, λ, and η, coefficients of dispersion diffusion, thermal conductivity, and viscosity; T and ΔT, temperature and its difference; c_p, specific heat at constant pressure; Q_x, heat flux; u, velocity of the gradient flow of the liquid; F, force; ΔK, change in the momentum; Δt, time; Re_{eq}, $Re_{\perp}$, $Re_{\parallel}$, and $Re_{\parallel,1}$, Reynolds numbers; Pr_M, dispersion Prandtl number; $\overline{a}$, value of the random quantity averaged over the ensemble; $D[a]$, dispersion of the quantity a; $W(a)$, probability density of the quantity a. Subscripts and superscripts: prime, relates to the quantities (pulsating in space) with a zero average value; x, y, z, $x + \Delta L$, $y + \Delta L$, $z + \Delta L$, $x + \Delta x$, $y + \Delta y$, and $z + \Delta z$, relate to the components along the coordinate axes; α and $\alpha + \Delta\alpha$, relate to the components along the arbitrary axis; $\perp$, relates to the components perpendicular to the main flow; $\parallel$, relates to the components in parallel to the main flow; M, relates to the quantities determined by macrodispersion; S, relates to the area; c, relates to the quantities determined by the concentration of the diffusing liquid; eq, equivalent; m, molecular; pore, pores; part, particle; p, isobaric; eff, effective.

REFERENCES

1. G. I. Taylor, *Proc. Roy. Soc.*, **A219**, No. 1133, 186–203 (1953).
2. G. I. Taylor, *Proc. Roy. Soc.*, **A223**, No. 1155, 446–468 (1954).
3. G. I. Taylor, *Proc. Roy. Soc.*, **A225**, No. 1163, 473–477 (1954).
4. R. Aris, *Proc. Roy. Soc.*, **A235**, No. 1200, 67–77 (1956).
5. P. G. Saffman, *J. Fluid Mech.*, **6**, No. 3, 321–349 (1959).
6. A. E. Scheidegger, *The Physics of Flow through Porous Media*, Toronto (1957).
7. *Development of Research in Filtration Theory in the USSR (1917–1967)* [in Russian], Moscow (1969).
8. V. N. Nikolaevskii, *Prikl. Mat. Mekh.*, **23**, No. 6, 1042–1050 (1959).
9. V. N. Nikolaevskii, *Mechanics of Porous and Cracked Media* [in Russian], Moscow (1984).
10. R. A. Greenkorn and D. P. Kessler, *Ind. Eng. Chem.*, **61**, No. 9, 14–32 (1969).
11. R. E. Haring and R. A. Greenkorn, *AIChE J.*, **16**, No. 3, 477–483 (1970).
12. L. I. Kheifets and A. V. Neimark, *Multiphase Processes in Porous Media* [in Russian], Moscow (1982).
13. A. M. Yaglom, *Usp. Mat. Nauk*, **7**, Issue 5 (51), 3–168 (1952).
14. A. S. Monin and A. M. Yaglom, *Statistical Hydromechanics* [in Russian], Pts. 1 and 2, Moscow (1967).
15. M. M. Mendel'son and M. I. Shvidler, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 181–184 (1967).
16. M. I. Shvidler, *Izv. Vyssh. Uchebn. Zaved., Geologiya Razvedka*, No. 5, 66–83 (1983).
17. M. I. Shvidler, *Statistical Hydrodynamics of Porous Media* [in Russian], Moscow (1985).
18. Yu. A. Buevich and A. I. Leonov, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 6, 167–175 (1967).
19. Yu. A. Buevich, Yu. A. Korneev, and I. N. Shchelchkova, *Inzh.-Fiz. Zh.*, **30**, No. 6, 979–985 (1976).
20. Yu. A. Buevich, *Inzh.-Fiz. Zh.*, **51**, No. 3, 450–458 (1986).
21. Yu. A. Buevich and V. A. Ustinov, *Inzh.-Fiz. Zh.*, **62**, No. 3, 396–403 (1992).
22. O. G. Martynenko and N. V. Pavlyukevich, *Inzh.-Fiz. Zh.*, **71**, No. 1, 5–18 (1998).

23. Yu. A. Kovalenko, in: *Proc. All-Union Seminar "Heat Transfer and Thermophysical Properties of Porous Materials"* [in Russian], Novosibirsk (1992), pp. 3–26.
24. R. I. Nigmatulin, *Fundamentals of the Mechanics of Heterogeneous Media* [in Russian], Moscow (1978).
25. R. I. Nigmatulin, *Dynamics of Multiphase Media* [in Russian], Pt. 1, Moscow (1987).
26. *Problems of Averaging and Construction of Continuous Models in the Mechanics of a Continuous Medium* [in Russian], Moscow (1980).
27. I. M. Lifshits, S. A. Grodeskul, and L. A. Pastur, *Introduction to the Theory of Disordered Systems* [in Russian], Moscow (1982).
28. N. S. Bakhvalov and G. P. Panasenko, *Averaging of the Processes in Periodic Media* [in Russian], Moscow (1984).
29. S. M. Kozlov, *Dokl. Akad. Nauk SSSR*, **241**, No. 5, 1016–1023 (1987).
30. M. I. Shvidler, *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 69–78 (1986).
31. V. I. Arnol'd and A. Avets, *Ergodic Problems of Classical Mechanics* [in Russian], Izhevsk (1999).
32. M. A. Gol'dshtik, *Transport Processes in a Granular Bed* [in Russian], Novosibirsk (1984).
33. A. V. Luikov, *Heat and Mass Transfer* [in Russian], Moscow (1972).
34. D. A. G. Bruggeman, *Ann. Phys.* (Leipzig), **24**, F. 5, No. 7, 636–664 (1935).
35. P. Ramon, *Field Theory: A Modern Primer* [Russian translation], Moscow (1984).
36. L. Raider, *Quantum Field Theory* [Russian translation], Moscow (1987).
37. R. J. Elliott, J. A. Krumhansl, and R. L. Leath, *Rev. Mod. Phys.*, **46**, No. 3, 18–34 (1974).
38. S. Kirkpatrik, *Rev. Mod. Phys.*, **45**, No. 4, 574–588 (1973).
39. B. I. Shklovskii and A. L. Éfros, *Electronic Properties of Doped Semiconductors* [in Russian], Moscow (1979).
40. D. S. Gaunt and M. F. Sykes, *J. Phys. A*, **16**, No. 4, 783–800 (1983).
41. I. M. Sokolov, *Usp. Fiz. Nauk*, **150**, Issue 2, 221–256 (1986).
42. A. V. Neimark, *Zh. Tekh. Fiz.*, **56**, No. 11, 834–842 (1981).
43. P. S. Grinchuk and N. V. Pavlyukevich, in: *Proc. IVth Minsk Int. Forum "Heat and Mass Transfer—MIF-2000,"* Minsk, May 22–26, 2000 [in Russian], Vol. 8, Minsk (2000), pp. 49–59.
44. O. Wiener, *Abh. Math.-Phys. Kl. Konigl-Sachs*, *Ges. D. Wiss.*, **32**, No. 6, 509–603 (1912).
45. R. Hill, *J. Mech. Phys. Solids*, **11**, No. 5, 357–362 (1963).
46. R. Hill, *J. Mech. Phys. Solids*, **12**, No. 4, 199–211 (1964).
47. A. M. Dykhne, *Zh. Ékp. Teor. Fiz.*, **52**, Issue 1, 264–270 (1967).
48. Z. Haschin and S. Strikman, *J. Appl. Phys.*, **33**, No. 10, 112–120 (1962).
49. K. A. Lur'e and A. V. Cherkaev, *Dokl. Akad. Nauk SSSR*, **264**, No. 5, 230–241 (1982).
50. G. N. Dul'nev and V. V. Novikov, *Inzh.-Fiz. Zh.*, **36**, No. 5, 901–910 (1979).
51. G. N. Dul'nev, *Coefficients of Transfer in Inhomogeneous Media* [in Russian], Leningrad (1979).
52. G. N. Dul'nev and Yu. P. Zarichnyak, *Heat Conduction of Mixtures and Composite Materials* [in Russian], Leningrad (1974).
53. G. N. Dul'nev and V. V. Novikov, *Inzh.-Fiz. Zh.*, **45**, No. 3, 443–451 (1983).
54. G. N. Dul'nev, D. P. Volkov, and V. I. Malarev, *Inzh.-Fiz. Zh.*, **56**, No. 2, 281–291 (1989).
55. G. N. Dul'nev and V. I. Malarev, *Inzh.-Fiz. Zh.*, **56**, No. 1, 141 (1989).
56. N. R. Dul'nev, Yu. P. Zarichnyak, and B. L. Muratov, *Inzh.-Fiz. Zh.*, **31**, No. 2, 278–283 (1976).
57. G. N. Dul'nev and V. I. Malarev, *Inzh.-Fiz. Zh.*, **59**, No. 3, 522–539 (1990).
58. G. N. Dul'nev, I. K. Meshkaskii, V. V. Novikov, and I. A. Sokolov, *Inzh.-Fiz. Zh.*, **37**, No. 2, 229–235 (1979).
59. G. R. Dul'nev, Yu. P. Zarichnyak, and V. V. Novikov, *Inzh.-Fiz. Zh.*, **32**, No. 1, 150–171 (1976).
60. V. I. Odelevskii, *Zh. Tekh. Fiz.*, **21**, Issue 6, 667–685 (1951).
61. A. M. Dykhne, *Zh. Éksp. Teor. Fiz.*, **59**, Issue 7, 110–119 (1970).
62. Yu. A. Buevich, *Prikl. Mat. Tekh. Fiz.*, No. 4, 57–66 (1973).
63. Yu. A. Buevich and Yu. A. Korneev, *Inzh.-Fiz. Zh.*, **31**, No. 4, 607–612 (1976).
64. Yu. A. Buevich and V. G. Markov, *Inzh.-Fiz. Zh.*, **36**, No. 5, 829–834 (1979).
65. M. I. Shvidler, *Zh. Éksp. Teor. Fiz.*, **80**, Issue 2, 34–42 (1981).
66. M. I. Shvidler, *Zh. Éksp. Teor. Fiz.*, **84**, Issue 3, 112–120 (1983).
67. A. F. Chudnovskii, *Thermophysical Characteristics of Disperse Materials* [in Russian], Moscow (1962).

68. L. L. Vasil'ev and S. A. Tanaeva, *Thermophysical Properties of Porous Materials* [in Russian], Minsk (1971).
69. A. V. Gorin and Yu. A. Kovalenko (eds.), *Proc. All-Union Seminar "Heat Transfer and Thermophysical Properties of Porous Materials"* [in Russian], Novosibirsk (1992).
70. V. V. Mandelbrot, *The Fractal Geometry of Nature*, Freeman, San Francisco (1982).
71. Ya. B. Zel'dovich and D. D. Sokolov, *Usp. Fiz. Nauk*, **146**, No. 3, 493–506 (1985).
72. B. M. Smirnov, *Usp. Fiz. Nauk*, **149**, No. 2, 177–219 (1986).
73. E. Feder, *Fractals* [Russian translation], Moscow (1991).
74. M. Barnsley, *Fractals Everywhere*, Academic Press, Boston (1988).
75. *Fractals in Physics, Proc. 6th Int. Symp. on Fractals in Physics* [in Russian], Moscow (1988).
76. A. Yu. Turygin and V. R. Chechetkin, *Zh. Éksp. Teor. Fiz.*, **98**, No. 1, 146–161 (1990).
77. V. V. Zosimov and L. M. Lyamshev, *Akust. Zh.*, **40**, No. 5, 709–736 (1994).
78. K. O. Sabdenov, *Fiz. Goren. Vzryva*, **31**, No. 6, 106–112 (1995).
79. H.-O. Peitgen and P. H. Richter, *The Beauty of Fractals: Images of Complex Dynamical Systems* [Russian translation], Moscow (1993).
80. M. Schroeder, *Fractals, Chaos, Power Laws* [Russian translation], Izhevsk (2000).
81. G. A. Edgar, *Measure, Topology, and Fractal Geometry*, Springer-Verlag, New York (1990).
82. S. V. Ostakh, in: *Proc. 2nd Russ. Nat. Conf. on Heat Transfer* [in Russian], Vol. 5, Moscow (1998), pp. 64–66.
83. R. M. Kronover, *Fractals and Chaos in Dynamic Systems. Fundamentals of the Theory* [in Russian], Moscow (2000).
84. R. R. Nigmatullin and N. N. Sutugin, *Inzh.-Fiz. Zh.*, **57**, No. 2, 291–298 (1989).
85. Yu. A. Kovalenko, V. A. Gruzdev, and Yu. A. Veslogzov, *Teplofiz. Vys. Temp.*, **33**, No. 3, 373–377 (1995).
86. *Heat-Transfer Processes in Power and Mechanical Engineering*, Abstracts of the Regional Interuniversity Seminar [in Russian], Voronezh (1995).
87. V. V. Shitov, in: V. V. Faleev (ed.), *Thermal-Power Engineering*, Collected Papers [in Russian], Voronezh (1995), pp. 71–74.
88. V. V. Shitov, in: V. V. Faleev (ed.), *Thermal-Power Engineering*, Collected Papers [in Russian], Voronezh (1996), pp. 15–20.
89. A. V. Mal'shakov, *Inzh.-Fiz. Zh.*, **62**, No. 3, 405–410 (1992).
90. L. S. Kokorev, V. I. Subbotin. V. N. Fedoseev, V. V. Kharitonov, and V. V. Voskoboinikov, *Teplofiz. Vys. Temp.*, **25**, No. 1, 92–97 (1987).
91. V. N. Fedoseev, V. I. Subbotin, and V. V. Kharitonov, *Teploénergetika*, No. 6, 61–64 (1987).
92. L. S. Kokorev, V. N. Fedoseev, V. V. Kharitonov, and V. V. Voskoboinikov, *New Approach to Calculation of Heat Transfer in Porous Media*, Preprint No. 024 of the Moscow Engineering Physics Institute [in Russian], Moscow (1986).
93. V. V. Atamanov, V. V. Kharitonov, and N. V. Yakutin, *Teplofiz. Vys. Temp.*, **34**, No. 4, 590–595 (1996).
94. D. Montgomery, *Res. Rept. Inst. Plasma Phys.*, Nagoya Univ., No. 670, 1–38 (1984).
95. A. P. Mirabel and A. S. Monin, *Usp. Mekh.*, **2**, Issue 3, 47–95 (1979).
96. *Turbulence Bidimentional*, *J. de Mec*., numero special, No. 2 (1988).
97. A. G. Bershadskii, *Prikl. Mat. Tekh. Fiz.*, No. 5, 109–115 (1988).
98. A. G. Bershadskii, *Zh. Éksp. Teor. Fiz.*, **96**, Issue 2, 625–631 (1989).
99. A. G. Bershadskii, *Prikl. Mat. Tekh. Fiz.*, No. 3, 49–51 (1990).
100. D. Palladin and A. Vul'piani, in: *Fractals in Physics* [in Russian], Moscow (1988), pp. 624–631.
101. M. É. Aérov, O. M. Todes, and D. A. Narinskii, *Apparatuses with a Stationary Granular Bed* [in Russian], Leningrad (1979).
102. B. M. Galitseiskii, Yu. N. Lapushkin, A. L. Lozhkin, and A. N. Ushakov, in: *Heat and Mass Transfer in Interaction of a Flow with a Surface*, Collected Scientific Papers of the Moscow Aviation Institute [in Russian], Moscow (1981), pp. 34–137.
103. A. L. Lozhkin and A. N. Ushakov, in: *Heat Transfer in Aviation Technology*, Collected Scientific Papers of the Moscow Aviation Institute [in Russian], Moscow (1984), pp. 19–22.
104. B. M. Galitseiskii, A. L. Lozhkin, and A. N. Ushakov, in: *Book of Abstr. of the Minsk Int. Forum "Heat and Mass Transfer—MIF-88"* [in Russian], Session 7, Minsk (1988), pp. 16–18.

105. A. L. Lozhkin, in: *Some Problems in Heat and Mass Exchange between Flows and Surfaces*, Collected Scientific Papers of the Moscow Aviation Institute [in Russian], Moscow (1986), pp. 69–70.
106. B. M. Galitseiskii, A. L. Lozhkin, and G. É. Solokhina, in: *Heat and Mass Transfer in Aircraft Engines*, Collected Scientific Papers of the Moscow Aviation Institute [in Russian], Moscow (1991), pp. 12–17.
107. B. M. Galitseiskii, A. L. Lozhkin, G. É. Solokhina, and A. N. Ushakov, in: *Heat Exchangers of Gas-Turbine Engines* [in Russian], Tr. TsIAM, No. 1282, Moscow (1991), pp. 22–28.
108. B. M. Galitseiskii, A. L. Lozhkin, and A. N. Ushakov, *Prom. Teplotekh.*, **13**, No. 1, 22–28 (1991).
109. B. M. Galitseiskii and A. L. Lozhkin, in: *Proc. 1st Russ. Nat. Conf. on Heat Transfer* [in Russian], Vol. 7, Moscow (1994), pp. 47–52.
110. B. M. Galitseiskii and A. L. Lozhkin, in: *Proc. IVth Minsk Int. Forum "Heat and Mass Transfer—MIF-2000,"* Minsk, May 22–26, 2000 [in Russian], Vol. 8, Minsk (2000), pp. 14–23.
111. B. M. Galitseisky and A. P. Mozhaev, in: *Proc. 3rd World Conf. Exp. Heat Transfer, Fluid Mechanics, and Thermodynamics,* Vol. 2, Honolulu, USA (1993), pp. 1569–1576.
112. V. M. Polyaev and A. P. Mozhaev, in: *Proc. Int. Symp. Turbulence, Heat and Mass Transfer*, Vol. 2, Lisbon, Portugal (1994), pp. 12.6.1–12.6.5.
113. V. M. Polyaev, B. M. Galitseiskii, and A. P. Mozhaev, *Vestn. MGTU, Mashinostroenie*, No. 3, 109–120 (1995).
114. A. P. Mozhaev and V. M. Polyaev, in: *Proc. 3rd Minsk Int. Forum "Heat and Mass Transfer—MIF-96,"* Minsk, May 20–24, 1996 [in Russian], Vol. 7, Minsk (1996), pp. 164–168.
115. V. M. Polyaev, A. P. Mozhaev, B. M. Galitseysky, and A. L. Lozhkin, *Exp. Thermal Fluid Sci.*, **12**, No. 4 (1996), pp. 426–432.
116. A. P. Mozhaev, in: *Proc. IVth Minsk Int. Forum "Heat and Mass Transfer—MIF-2000,"* Minsk, May 22–26, 2000 [in Russian], Vol. 8, Minsk (2000), pp. 3–13.
117. A. P. Mozhaev, *Inzh.-Fiz. Zh.*, **74**, No. 5, 196–200 (2001).
118. Yu. V. Polezhaev and A. A. Shishkov, *Gasdynamic Tests of a Thermal Shield. Handbook* [in Russian], Moscow (1992).
119. Yu. V. Polezhaev and F. B. Yurevich, *Thermal Protection* [in Russian], Moscow (1976).
120. S. A. Druzhinin and A. I. Leont'ev, *Inzh.-Fiz. Zh.*, **10**, No. 4, 479–481 (1966).
121. A. I. Leont'ev, B. P. Mironov, and P. P. Lugovskoi, *Inzh.-Fiz. Zh.*, **10**, No. 4, 447–451 (1966).
122. A. I. Leont'ev, B. P. Mironov, and A. V. Fafurin, *Teplofiz. Vys. Temp.*, **7**, No. 6, 1134–1140 (1969).
123. S. S. Kutateladze, I. G. Malenkov, and S. L. Vitkovskii, *Teplofiz. Vys. Temp.*, **17**, No. 2, 333–336 (1979).
124. S. S. Kutateladze and A. I. Leont'ev, *Turbulent Boundary Layer of an Incompressible Gas* [in Russian], Novosibirsk (1962).
125. S. S. Kutateladze and A. I. Leont'ev, *Heat and Mass Transfer and Friction in a Turbulent Boundary Layer* [in Russian], Moscow (1985).
126. R. G. Bogoyavlenskii, *Hydrodynamics and Heat Transfer in High-Temperature Nuclear Reactors with Spherical Fuel Elements* [in Russian], Moscow (1978).
127. A. V. Afanas'ev, L. L. Kalishevskii, and V. G. Krapivtsev, in: *Current Problems of Hydrodynamics and Heat Transfer in the Elements of Power Plants and Cryogenic Technology*, Collected Scientific Papers [in Russian], Moscow (1988), pp. 10–16.
128. A. V. Afanas'ev, L. L. Kalishevskii, and V. G. Krapivtsev, in: *Book of Abstr. of the Minsk Int. Forum "Heat and Mass Transfer—MIF-98*" [in Russian], Session 5, Minsk (1998), pp. 7–9.
129. A. V. Afanas'ev, A. V. Istomin, and V. G. Krapivtsev, in: *Current Problems of Hydrodynamics and Heat Transfer in the Elements of Power Plants and Cryogenic Technology*, Collected Scientific Papers [in Russian] (989), pp. 92–97.
130. R. E. Collins, *Flow of Fluids through Porous Materials*, Reinhold, New York (1961).
131. I. Z. Fisher, *Statistical Theory of a Liquid* [in Russian], Moscow (1961).
132. I. V. Savel'ev, *A Course in General Physics* [in Russian], Vol. 1, Moscow (1966).
133. V. F. Nozdrev and A. A. Senkevich, *A Course in Statistical Physics* [in Russian], Moscow (1969).
134. A. N. Kolmogorov and S. V. Fomin, *Elements of the Theory of Functions and Functional Analysis* [in Russian], Moscow (1976).